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Design of Complex-valued Hopfield Associative Memory Based on Prespecified Attractive Domain

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Abstract: This paper proposes a connection weighting scheme of a complex-valued Hopfield neural network for associative memory constrained by given attractive domain. Both equilibrium conditions and stability analysis results are used in the synthesis procedure. We solve the equilibrium equation by singular value decomposition technique and obtain a general solution of the connection weight matrix with a free sub-matrix. Such general solution and the parameter matrix corresponding to the given attractive domain are contained in the inequations which are derived from stability analysis and can be represented as linear matrix inequations (LMIs). The connection weighting solution of such LMIs can guarantee stability and attractability of the network simultaneously. A simulation example of a 3-dimension complex-valued Hopfield neural network shows the proposed synthesis method. The simulation results demonstrate the attractive ability of two complex-valued vectors in the prespecified attractive domain.

Keywords: complex-valued Hopfield neural network; associative memory; stability analysis; attractive domain

1. Introduction

In resent years, the research and application of complex-valued neural networks, because of their ability in dealing with complex-valued data directly, have got growth [1]. In this paper we discuss a class of complex-valued associative memory as the application of complex-valued Hopfield neural network (CHNN).

It is known that in a well designed Hopfield associative memory, each desired memory pattern should be stable and attractive, i.e. an asymptotically stable equilibrium point of the system, and the corresponding attractive domain should be as large as possible. However, among the three conventional learning rules for the complex-valued Hopfield network: Hebb rule [2], projection rule [3] and gradient descent learning rule [4], neither stability nor attractability is considered.

To guarantee the stability of real-valued Hopfield neural network (RHNN), A.N.Michel synthesized a discrete time network by equilibrium equation, where an adjustable gain of the activation function was utilized to satisfy the asymptotically stable condition derived from Lyapunov method [5]. In reference [6], a class of generalized continuous time network was designed by solving equilibrium equation with singular value decomposition technique, and the stability properties of the given equilibrium points were analyzed in terms of the eigenvalues of Jacobian matrix of energy function. Y.Kuroe *et al.* proposed complex-valued energy function and extended the synthesis method of reference [6] to complex-valued domain [7].

In the above synthesis approaches (whether for RHNN or for CHNN) the attractiveness is not a design condition. At the same time, attractive ability is essential for associative memory. And in some practical applications, there are special requirements of the attractive domains, such as the size or the shape of the domain. Considering the size of attractive domain, Muezzinoglu *et al.*designed real-valued and complex-valued Hopfield associative memory in the related work [8,9]. Their methods were to construct energy landscape and solve homogenous linear inequalities, which comes from the definition that energy function of each equilibrium point is the strict local minimum.

In present research of associative memories, the stability analysis based on Lyapunov theory has been widely used to evaluate and enlarge the attractive domains [10]. To store the desired complex-valued pattern vectors and recall their neighbor vectors in the prespecified attractive domain, this paper utilizes the Lyapunov synthesis method for complex-valued Hopfield associative memory. At first we solve the equilibrium equations of the network by singular value decomposition and obtain a normal solution of connection weight matrix in which there is a free submatrix undetermined. Then, by Lyapunov theory, the asymptotically stability conditions are obtained which contain parameter matrix of the given attractive domain and can be expressed as LMIs. Finally, substituting the normal weight solution into the LMIs, we get a feasible solution of the undetermined submatrix, and thus accomplish the synthesis of the desired CVNN.

In the following, the imaginary unit is denoted by $i(i^2=-1)$. For complex-valued matrix (vector or number) $\mathbf{A}\in\mathbb{C}^{n\times m}(\mathbf{a}\in\mathbb{C}^n, a\in\mathbb{C})$, its real and imaginary parts are denoted by$\mathbf{A}^R(\mathbf{a}^R,a^R)$ and $\mathbf{A}^I(\mathbf{a}^I,a^I)$respectively. $\mathbf{A}^T$ denotes the transpose of $\mathbf{A}$ and $\mathbf{A}^*$ denotes the conjugate transpose of $\mathbf{A}$. The notion $\mathbf{X}\geq\mathbf{Y}(\mathbf{X}>\mathbf{Y})$, where $\mathbf{X}$ and $\mathbf{Y}$ are symmetric matrix, means that $\mathbf{X}-\mathbf{Y}$ is positive semidefinite (positive definite). * refers to the element below the main diagonal of a symmetric block matrix.

2. Model of CHNN

2.1 Description of the model

Consider a class of complex-valued Hopfield neural network (CHNN). The CHNN has n neurons and the dynamics of the ith neuron is described by following algorithm:

$$\begin{cases} \frac{du_i(t)}{dt}=-c_iu_i(t)+\sum\limits_{j=1}^{n}a_{ij}V_j(t)+I_i \\ V_i(t)=f_i(u_i(t)) \end{cases} \tag{1}$$

where $u_i(t)\in\mathbb{C}, V_i(t)\in\mathbb{C}$ and $I_i\in\mathbb{C}$ are the state, the output and the threshold value of the ith neuron at time t ,respectively,$a_{ij}\in\mathbb{C}$ is the connection weight from the jth neuron to the ith neuron, $C_i>0$ is time constant of the ith neuron,$f_i(\cdot)$ is the activation function which is a nonlinear complex-valued function $(f_i:\mathbb{C}\to\mathbb{C})$.

Note that the neural network described by (1) is a direct complex-valued extension of the real-valued neural network of Hopfield type. In the real-valued neural network for associative memory, the activation function is usually chosen to be a smooth and bounded analytic function such as sigmoid function. In the complex region, however, such a condition is not suitable for complex activation function in that according to Liouville's theorem,if $f(u)$ is analytic at all $u\in\mathbb{C}$ and is bounded, then $f(u)$ is a constant function[11]. For this reason, we choose a class of complex-valued activation function described in [11]:

$$f_i(u_i)=f_i^R(u_i^R)+if_i^I(u_i^I) \tag{2}$$

where $f_i^R(f_i^I)$ is bounded, monotone nondecreasing, continuously differentiable with respect to $u_i^R(u_i^I)$, that is, $\partial f_i^R/\partial u_i^R>0, \partial f_i^I/\partial u_i^I>0$.

Y.Kuroe *et al.* obtained in [11] that with such complex-valued activation function and Hermitian connection weight matrix, the continuous model of CHNN (1) has an energy function.

We relax the continuously differentiable to locally Lipschitz conditions with Lipschitz constants $\overline{G_i}$ and $\overline{H_i}$, that is, there exist two constant matrices: $\overline{\mathbf{G}}=diag(\overline{G}_1,\overline{G}_2,\ldots,\overline{G}_n), \overline{\mathbf{H}}=diag(\overline{H}_1,\overline{H}_2,\ldots,\overline{H}_n)$, where $0\leq\overline{G},\overline{H}<+\infty$, such that

$$0\leq\frac{f_i^R(x_i)-f_i^R(y_i)}{x_i-y_i}\leq\overline{G}_i, 0\leq\frac{f_i^I(x_i)-f_i^I(y_i)}{x_i-y_i}\leq\overline{H}_i \tag{3}$$

for all $x_i,y_i\in B_\delta(x^e)(x_i\neq y_i, i=1,2,\ldots,n)$ where $B_\delta(x^e) =\{x=[x_1,x_2,\ldots,x_n]^T\in R^n:\|x-x^e\|<\delta\}$.

2.2 Translation of the equilibrium

Without loss of generality, we suppose the CHNN (1) has equilibrium $\mathbf{u}^e=[u_1^e,u_2^e,\ldots,u_n^e]^T\in C^n$ not locating at the origin $(u_i=0+i0, i=1,2,\ldots,n)$. By definition, $\mathbf{u}^e$ satisfies the equation

$$0=-c_iu_i^e+\sum_{i=1}^{n}a_{ij}f_j(u_j^e)+I_i \quad (i=1,2,\ldots,n) \tag{4}$$

To discuss the stability and the attraction of network (1), we need shift the equilibrium point to the origin by using the transformation $y_i(t) = u_i(t) - u_i^e$. Then network (1) can be transformed into the form

$$\frac{dy_i(t)}{dt} = -c_i y_i(t) + \sum_{j=1}^{n} a_{ij} g_j(y_j(t)) \quad (5)$$

where $g_j(y_j(t)) = f_j(y_j(t) + u_j^e) - f_j(u_j^e)$

From (2) we can rewrite $g_j(y_j)$ explicitly as

$$\begin{aligned} g_j(y_j) &= g_j^R(y_j^R) + i g_j^I(y_j^I) \\ &= [f_j^R(y_j^R + (u_j^e)^R) - f_j^R(u_j^e)^R] + \\ &\quad i[f_j^I(y_j^I + (u_j^e)^I) - f_j^I((u_j^e)^I)] \end{aligned}$$

In the following discussion we denote $y_j(t)$ as y_j and $g_j(y_j)$ as g_j for simplifying description. Thus

$$\begin{aligned} \frac{dy_i}{dt} &= \frac{dy_i^R}{dt} + i\frac{dy_i^I}{dt} \\ &= [-c_i y_i^R + \sum_{j=1}^{n}(a_{ij}^R g_j^R - a_{ij}^I g_j^I)] + \quad (6) \\ &\quad i[-c_i y_i^I + \sum_{j=1}^{n}(a_{ij}^I g_j^R + a_{ij}^R g_j^I)] \end{aligned}$$

The corresponding matrix vector form is

$$\begin{aligned} \frac{d\mathbf{y}}{dt} &= \frac{d\mathbf{y}^R}{dt} + i\frac{d\mathbf{y}^I}{dt} \\ &= (-\mathbf{C}\mathbf{y}^R + \mathbf{A}^R\mathbf{g}^R - \mathbf{A}^I\mathbf{g}^I) + \quad (7) \\ &\quad i(-\mathbf{C}\mathbf{y}^I + \mathbf{A}^I\mathbf{g}^R + \mathbf{A}^R\mathbf{g}^I) \end{aligned}$$

where $\mathbf{y} = [y_1, \ldots, y_n]^T = \mathbf{y}^R + i\mathbf{y}^I, \mathbf{C} = diag(c_1, \ldots, c_n)$, $\mathbf{A} = (a_{ij})_{n\times n} = \mathbf{A}^R + i\mathbf{A}^I, \mathbf{g}^R = [g_1^R, \ldots, g_n^R]^T$ and $\mathbf{g}^I = [g_1^I, \ldots, g_n^I]^T$

By local Lipschitz condition in (3), it is easy to obtain the following inequalities:

$$(\mathbf{g}^R)^T\mathbf{S}\mathbf{g}^R \le (\mathbf{y}^R)^T\overline{\mathbf{G}}\mathbf{S}\overline{\mathbf{G}}\mathbf{y}^R \quad (8)$$

$$(\mathbf{g}^I)^T\mathbf{T}\mathbf{g}^I \le (\mathbf{y}^I)^T\overline{\mathbf{H}}\mathbf{T}\overline{\mathbf{H}}\mathbf{y}^I \quad (9)$$

where $\mathbf{S}, \mathbf{T} \in R^{n\times n}$ can be any nonnegative diagonal matrices.

2.3 Stability analysis of CHNN

For the purpose of associative memory, each memory vector that we desire for the CHNN (1) to store must be stable and attractive, i.e., local asymptotic stable equilibrium of the network. Therefore it is necessary to analyze the stability condition of the network.

Theorem 1 For given real-valued $\mathbf{P} > 0, \mathbf{Q} > 0$ consider the set

$$\begin{aligned} \mathbb{M}(P,Q) := \quad &\{u \in C^n : (u^R)^T P u^R \le 1, \\ &(u^I)^T Q u^I \le 1\} \end{aligned} \quad (10)$$

if there exist network (7) and diagonal matrices and of compatibly dimensions such that

$$\begin{aligned} -2\mathbf{PC} + \mathbf{PA}^R\mathbf{S}^{-1}(\mathbf{A}^R)^T\mathbf{P} + \mathbf{PA}^I\mathbf{T}^{-1}(\mathbf{A}^I)^T\mathbf{P} \\ + 2\overline{\mathbf{G}}\mathbf{S}\overline{\mathbf{G}} < \mathbf{0} \end{aligned} \quad (11)$$

and

$$\begin{aligned} -2\mathbf{QC} + \mathbf{QA}^I\mathbf{S}^{-1}(\mathbf{A}^I)^T\mathbf{Q} + \mathbf{QA}^R\mathbf{T}^{-1}(\mathbf{A}^R)^T\mathbf{Q} \\ + 2\overline{\mathbf{H}}\mathbf{T}\overline{\mathbf{H}} < \mathbf{0} \end{aligned} \quad (12)$$

namely the matrix inequalities

$$\begin{bmatrix} -2\mathbf{PC} + 2\overline{\mathbf{G}}\mathbf{S}\overline{\mathbf{G}} & \mathbf{PA}^R & \mathbf{PA}^I \\ * & -\mathbf{S} & 0 \\ * & * & -\mathbf{T} \end{bmatrix} < 0 \quad (13)$$

$$\begin{bmatrix} -2\mathbf{QC} + 2\overline{\mathbf{H}}\mathbf{T}\overline{\mathbf{H}} & \mathbf{QA}^I & \mathbf{QA}^R \\ * & -\mathbf{S} & 0 \\ * & * & -\mathbf{T} \end{bmatrix} < 0 \quad (14)$$

then the origin of network (7) is locally asymptotically stable, the set $\mathbb{M}(\mathbf{P}, \mathbf{Q})$ is an attractive domain. And the trivial solution $\mathbf{u} \equiv \mathbf{u}^e$ of network (1) is locally asymptotically stable, the set

$$\begin{aligned} \mathbb{M}'(\mathbf{P},\mathbf{Q}) := \{\mathbf{u} \in C^n : (\mathbf{u^R} - (\mathbf{u^e})\mathbf{R})^\mathbf{T}\mathbf{P}(\mathbf{u^R} - (\mathbf{u^e})^\mathbf{R}) \le \mathbf{1}, \\ (\mathbf{u^I} - (\mathbf{u^e})^\mathbf{I})^\mathbf{T}\mathbf{Q}(\mathbf{u^I} - (\mathbf{u^e})^\mathbf{I}) \le \mathbf{1}\} \end{aligned}$$

is a domain of attraction of equilibrium point $\mathbf{u}^e$ for network (1)

Proof Given $\mathbf{P} > 0$ and $\mathbf{Q} > 0$, consider a quadratic Lyapunov function with complex-valued variables

$$v(\mathbf{y}(t)) = (\mathbf{y}^R(t))^T\mathbf{P}\mathbf{y}^R(t) + (\mathbf{y}^I(t))^T\mathbf{Q}\mathbf{y}^I(t) \quad (15)$$

where $\mathbf{y}(t) = \mathbf{y}^R(t) + i\mathbf{y}^I(t), \mathbf{y}(t) \in \mathbb{M}(\mathbf{P}, \mathbf{Q})$.

Firstly, it is easy to guarantee that $v(\mathbf{y}(t))$ is bounded. The derivative of $v(\mathbf{y}(t))$ along the trajectories of system(7) is given by

$$\begin{aligned} \dot{v}(\mathbf{y}) &= 2(\mathbf{y}^R)^T\mathbf{P}\dot{\mathbf{y}}^R + 2(\mathbf{y}^I)^T\mathbf{Q}\dot{\mathbf{y}}^I \\ &= 2(\mathbf{y}^R)^T\mathbf{P}(-\mathbf{C}\mathbf{y}^R + \mathbf{A}^R\mathbf{g}^R - \mathbf{A}^I\mathbf{g}^I) + \\ &\quad 2(\mathbf{y}^I)^T\mathbf{Q}(-\mathbf{C}\mathbf{y}^I + \mathbf{A}^I\mathbf{g}^R + \mathbf{A}^R\mathbf{g}^I) \\ &= -(\mathbf{y}^R)^T\mathbf{PC}\mathbf{y}^R + (\mathbf{y}^R)^T\mathbf{PA}^R\mathbf{g}^R - \\ &\quad (\mathbf{y}^R)^T\mathbf{PA}^I\mathbf{g}^I - 2(\mathbf{y}^I)^T\mathbf{QC}\mathbf{y}^I + \\ &\quad 2(\mathbf{y}^I)^T\mathbf{QA}^I\mathbf{g}^R + 2(\mathbf{y}^I)^T\mathbf{QA}^R\mathbf{g}^I \end{aligned}$$

By applying inequalities

$$\mathbf{\Sigma}_1^T\mathbf{\Sigma}_2 + \mathbf{\Sigma}_2^T\mathbf{\Sigma}_1 \le \mathbf{\Sigma}_1^T\mathbf{\Sigma}_3\mathbf{\Sigma}_1 + \mathbf{\Sigma}_2^T\mathbf{\Sigma}_3^{-1}\mathbf{\Sigma}_2 \quad (16)$$

and inequalities (8) , (9) , it will lead to

$$\begin{aligned}&2(\mathbf{y}^R)^T\mathbf{P}\mathbf{A}^R\mathbf{g}^R\\&=(\mathbf{y}^R)^T\mathbf{P}\mathbf{A}^R\mathbf{g}^R+(\mathbf{g}^R)^T(\mathbf{A}^R)^T\mathbf{P}\mathbf{y}^R\\&\leq(\mathbf{y}^R)^T\mathbf{P}\mathbf{A}^R\mathbf{S}^{-1}(\mathbf{A}^R)^T\mathbf{P}\mathbf{y}^R+(\mathbf{g}^R)^T\mathbf{S}\mathbf{g}^R\\&\leq(\mathbf{y}^R)^T\mathbf{P}\mathbf{A}^R\mathbf{S}^{-1}(\mathbf{A}^R)^T\mathbf{P}\mathbf{y}^R+(\mathbf{y}^R)^T\overline{\mathbf{G}}\mathbf{S}\overline{\mathbf{G}}\mathbf{y}^R.\end{aligned}\tag{17}$$

$$\begin{aligned}-2(\mathbf{y}^R)^T\mathbf{P}\mathbf{A}^I\mathbf{g}^I\leq(\mathbf{y}^R)^T\mathbf{P}\mathbf{A}^I\mathbf{T}^{-1}(\mathbf{A}^I)^T\mathbf{P}\mathbf{y}^R\\+(\mathbf{y}^I)^T\overline{\mathbf{H}}\mathbf{T}\overline{\mathbf{H}}\mathbf{y}^I\end{aligned}\tag{18}$$

$$\begin{aligned}2(\mathbf{y}^I)^T\mathbf{Q}\mathbf{A}^I\mathbf{g}^R\leq(\mathbf{y}^I)^T\mathbf{Q}\mathbf{A}^I\mathbf{S}^{-1}(\mathbf{A}^I)^T\mathbf{Q}\mathbf{y}^I\\+(\mathbf{y}^R)^T\overline{\mathbf{G}}\mathbf{S}\overline{\mathbf{G}}\mathbf{y}^R\end{aligned}\tag{19}$$

$$\begin{aligned}2(\mathbf{y}^I)^T\mathbf{Q}\mathbf{A}^R\mathbf{g}^I\leq(\mathbf{y}^I)^T\mathbf{Q}\mathbf{A}^R\mathbf{T}^{-1}(\mathbf{A}^R)^T\mathbf{Q}\mathbf{y}^R\\+(\mathbf{y}^I)^T\overline{\mathbf{H}}\mathbf{T}\overline{\mathbf{H}}\mathbf{y}^I\end{aligned}\tag{20}$$

With (16) – (20), we can obtain

$$\dot{v}(\mathbf{y})\leq(\mathbf{y}^R)^T\mathbf{\Gamma}_1\mathbf{y}^R+(\mathbf{y}^I)^T\mathbf{\Gamma}_2\mathbf{y}^I\tag{21}$$

where

$\mathbf{\Gamma}_1=-2\mathbf{PC}+\mathbf{P}\mathbf{A}^R\mathbf{S}^{-1}(\mathbf{A}^R)^T\mathbf{P}+\mathbf{P}\mathbf{A}^I\mathbf{T}^{-1}(\mathbf{A}^I)^T\mathbf{P}+2\overline{\mathbf{G}}\mathbf{S}\overline{\mathbf{G}}$

$\mathbf{\Gamma}_2=-2\mathbf{QC}+\mathbf{Q}\mathbf{A}^I\mathbf{S}^{-1}(\mathbf{A}^I)^T\mathbf{Q}+\mathbf{Q}\mathbf{A}^R\mathbf{T}^{-1}(\mathbf{A}^R)^T\mathbf{Q}+2\overline{\mathbf{H}}\mathbf{T}\overline{\mathbf{H}}$

By condition (11) and (12) in Theorem 1, it is easy to see

$$\Gamma_1<0,\Gamma_2<0\tag{22}$$

Using Schur Complement, we can rewrite (22) as linear matrix inequalities in form of (13) and (14), respectively. Then with inequalities (21) and (22), we get

$$\dot{v}(\mathbf{y})<0\tag{23}$$

Thus the origin of CHNN (7) is locally asymptotically stable and the set $\mathbb{M}(\mathbf{P},\mathbf{Q})$ is an attractive domain.

For the network in (1), by coordinate transformation, the set $\mathbb{M}'(\mathbf{P},\mathbf{Q})$ is a domain of attraction of equilibrium point $\mathbf{u}^e$ for the network in (1). The proof is complete.

3. Parameter designation of CHNN

3.1 Description of designation problem

To store a given complex-valued pattern vector set in CHNN (1) and guarantee each pattern can attract the vectors in a prespecified neighbor domain, in this paper we design the connection weight matrix **A** and the threshold matrix **I**, while the time constant matrix **C** is predefined to control the evolution speed of the system.

As mentioned above, to store patterns in a Hopfield neural network, the patterns must be the equilibrium points of the network. Furthermore, to let the network recall the patterns from the corrupted or incomplete patterns corresponding to them, the equilibrium points must be attractive, that is, locally asymptotically stable.

Thus in the following synthesis procedure both the equilibrium constrains and stable conditions are applied

3.2 Equilibrium constrains

Suppose a set of complex-valued pattern vectors is $\mathbf{\Lambda}=\{\mathbf{V}^1,\ldots,\mathbf{V}^r,\ldots,\mathbf{V}^{m+1}\}$, where $\mathbf{V}^r=[V_1^r,\ldots,V_n^r]\in\mathbb{C}^n, r=1,\ldots,m+1$

In order to store the $m+1$ vectors, each vector $\mathbf{V}^r$ should be an equilibrium point of the CHNN (1), which means the vector must satisfy equilibrium equation (4). So we get

$$0=-c_iu_i^r+\sum_{i=1}^{n}a_{ij}V_j^r+I_i\tag{24}$$

$$V_i^r=f_i(u_i^r)=f_i^R((u_i^r)^R)+if_i^I((u_i^r)^I)\tag{25}$$

for $i=1,\ldots,n,\quad r=1,\ldots,m+1$, where $\mathbf{V}^r$ and $\mathbf{u}^r=[u_1^r,\ldots,u_n^r]^T$ are the corresponding rth output and input pattern vectors, respectively.

Suppose f_i^R and f_i^I are all invertible, a sample calculation

$$u_i^r=(u_i^r)^R+i(u_i^r)^I=f_i^{R^{-1}}((V_i^r)^R)+if_i^{I^{-1}}((V_i^r)^I)\tag{26}$$

will yield $\mathbf{u}^r$ for any $\mathbf{V}^r$.

In terms of $\mathbf{u}^r$ and $\mathbf{V}^r$, the equilibrium constrains (24) can be represented in a matrix form:

$$\begin{cases}0&=&-\mathbf{C}\mathbf{u}^1+\mathbf{A}\mathbf{V}^1+\mathbf{I}\\\vdots&&\\0&=&-\mathbf{C}\mathbf{u}^{m+1}+\mathbf{A}\mathbf{V}^{m+1}+\mathbf{I}\end{cases}\tag{27}$$

By defining

$$\mathbf{u}=[\mathbf{u}^2-\mathbf{u}^1\vdots\cdots\vdots\mathbf{u}^{m+1}-\mathbf{u}^1],\tag{28}$$

$$\mathbf{V}=[\mathbf{V}^2-\mathbf{V}^1\vdots\cdots\vdots\mathbf{V}^{m+1}-\mathbf{V}^1],\tag{29}$$

Equation(27) are transformed to

$$\mathbf{Cu}=\mathbf{AV},\tag{30}$$

$$\mathbf{I}=\mathbf{C}\mathbf{u}^{m+1}-\mathbf{A}\mathbf{V}^{m+1}.\tag{31}$$

According to conventional projection rule, the solution of weight matrix $\mathbf{A}$ is calculated from (30), that is

$$\mathbf{A} = \mathbf{CuV}^{+}, \tag{32}$$

where $\mathbf{V}^{+}$ is the Pseudoinverse of $\mathbf{V}$. It is obvious that the weight matrix $\mathbf{A}$ is not Hermitian, thus the stability of the pattern vectors $\mathbf{V}^{r}$ is undetermined from analysis in [11].

In fact, the solution of (30) is generally not unique. To obtain a Hermitian matrix $\mathbf{A}$, that is $\mathbf{A} = \mathbf{A}^{*}$, we employ singular value decomposition technique and derive a normal solution of (30) as follows.

Suppose the singular value decomposition of $\mathbf{V}$ is

$$\mathbf{V} = \mathbf{L\Sigma M}^{*}, \tag{33}$$

where $\mathbf{L}$ is a $n\times n$ complex-valued unitary matrix, $\mathbf{M}$ is a $m\times m$ complex-valued unitary matrix, and $\mathbf{\Sigma}$ is a $n\times m$ real-valued diagonal matrix with singular values of $\mathbf{V}$ and zero as its diagonal elements. Generally the number of neurons (n) is greatly larger than the number of memory patterns $(m+1)$, and the patterns are not linear dependent. Thus, it is reasonable to suppose the rank of matrix $\mathbf{\Sigma}$ is m and denote $\mathbf{\Sigma}$ by

$$\mathbf{\Sigma} = \begin{bmatrix} \mathbf{\Sigma}_1 \\ \mathbf{0} \end{bmatrix}. \tag{34}$$

where $\Sigma_1 = diag(\sigma_1,\ldots,\sigma_m), \sigma_1 \geq \sigma_2 \geq \cdots \geq \sigma_m > 0$ are the singular values of $\mathbf{V}$

Substituting (33) into (30), we get $\mathbf{A} = (\mathbf{L}\ \mathbf{\Sigma}\ \mathbf{M}^{*}) = \mathbf{Cu}$ and

$$\mathbf{L}^{*}\mathbf{AL\Sigma} = \mathbf{L}^{*}\mathbf{CuM} \triangleq \begin{bmatrix} \mathbf{C}_1 \\ \mathbf{C}_2 \end{bmatrix}. \tag{35}$$

where $\mathbf{C}_1$ is a complex-valued matrix and $\mathbf{C}_2$ is a $(n-m)\times m$ complex-valued matrix. Also, let us define a real-valued square matrix with dimension $n\times n$

$$\mathbf{R} = \begin{bmatrix} \mathbf{\Sigma}_1 & 0 \\ 0 & \mathbf{E} \end{bmatrix}, \tag{36}$$

where $\mathbf{E}$ denotes a $(n-m)\times(n-m)$ identity matrix. It is obvious that $\mathbf{R}$ is $n\times n$ a real-valued symmetric matrix, i.e.$\mathbf{R}^{*} = \mathbf{R}$. Next, we solve

Next, we solve

$$\begin{aligned} \mathbf{R}^{*}\mathbf{L}^{*}\mathbf{ALR} &= \mathbf{RL}^{*}\mathbf{AL}\begin{bmatrix} \mathbf{\Sigma}_1 & 0 \\ 0 & \mathbf{E} \end{bmatrix} \\ &= \mathbf{R}\begin{bmatrix} \mathbf{C}_1 & \mathbf{\Phi} \\ \mathbf{C}_2 & \mathbf{\Psi} \end{bmatrix} \\ &= \begin{bmatrix} \mathbf{\Sigma}_1\mathbf{C}_1 & \mathbf{\Sigma}_1\mathbf{\Phi} \\ \mathbf{C}_2 & \mathbf{\Psi} \end{bmatrix} \end{aligned} \tag{37}$$

where $\mathbf{\Phi}$ is a $n\times(n-m)$ parameter matrix and $\mathbf{\Psi}$ is a $(n-m)\times(n-m)$ parameter matrix. Since $\mathbf{A}$ is Hermitian, to ensure a solution of (37), it is needed that

$$\mathbf{\Sigma}_1\mathbf{C}_1 = \mathbf{C}_1^{*}\mathbf{\Sigma}_1^{*} = \mathbf{C}_1^{*}\mathbf{\Sigma}_1, \tag{38}$$

and $\mathbf{\Phi} = \mathbf{\Sigma}_1^{-1}\mathbf{C}_2^{*}, \mathbf{\Psi} = \mathbf{\Psi}^{*}$

Remark 1. Constraint (38) is a necessary condition for equilibrium equations (30) to have a solution. It is easy to prove that $\mathbf{\Sigma}_1\mathbf{C}_1 = \mathbf{C}_1^{*}\mathbf{\Sigma}_1$ condition (38) is equivalent to

$$\mathbf{V}^{*}\mathbf{u} = \mathbf{u}^{*}\mathbf{V}, \tag{39}$$

(See also in reference [6]). So we suppose the desired memory pattern vector set $\mathbf{\Lambda}$ satisfy condition (39) in the following designation scheme for CHNN.

If condition (39) is satisfied, then (30) has a solution

$$\mathbf{A} = \mathbf{LDL}^{*} \tag{40}$$

where

$$\mathbf{D} = \begin{bmatrix} \mathbf{C}_1\mathbf{\Sigma}_1^{-1} & \mathbf{\Phi} \\ \mathbf{C}_1\mathbf{\Sigma}_1^{-1} & \mathbf{\Psi} \end{bmatrix}. \tag{41}$$

Remark 2. Solution (40) is a normal solution of equilibrium (30) from the standpoint that we can choose the conjugate symmetric matrix $\mathbf{\Psi}$ randomly while the weight matrix $\mathbf{A}$ from (40) can satisfy the equilibrium constraint (30). On the other hand, if we loose symmetric constrain for weight matrix $\mathbf{A}$ and let $\mathbf{\Phi} = 0$ and $\mathbf{\Psi} = 0$, it can be proved that the solution (40) is the Pseudoinverse solution (32) in the conventional projection rule. Thus we can conclude that (32) is a special case of (40)

Next, we set $\mathbf{\Psi} = \alpha\mathbf{E}$ to simplify the following designation, where $\mathbf{E}$ is an identity matrix and α is a parameter to be determined. By denoting $\mathbf{L} = [\mathbf{L}_1 \quad \mathbf{L}_2]$, where, $\mathbf{L}_1 \in C^{n\times m}, \mathbf{L}_2 \in C^{n\times(n-m)}$, and $\mathbf{D}_1 = \mathbf{C}_1\mathbf{\Sigma}_1^{-1}$, $\mathbf{D}_2 = \mathbf{C}_2\mathbf{\Sigma}_1^{-1} = \mathbf{\Phi}^{*}$, formula (40) can be rewrote as

$$\mathbf{A} = [\mathbf{L}_1 \quad \mathbf{L}_2]\begin{bmatrix} \mathbf{D}_1 & \mathbf{D}_2^{*} \\ \mathbf{D}_2 & \alpha\mathbf{E} \end{bmatrix}\begin{bmatrix} \mathbf{L}_1^{*} \\ \mathbf{L}_2^{*} \end{bmatrix} = \mathbf{A}^{R} + i\mathbf{A}^{I} \tag{42}$$

where

$$\begin{aligned} \mathbf{A}^{R} &= (\mathbf{L_1D_1L_1^{*}} + \mathbf{L_2D_2L_1^{*}} + \mathbf{L_1D_2^{*}L_2^{*}})^{R} \\ &\quad + \alpha(\mathbf{L_2L_2^{*}})^{R} \\ &\triangleq \mathbf{X}_{AR1} + \alpha\mathbf{X}_{AR2} \end{aligned} \tag{43}$$

$$\begin{aligned} \mathbf{A}^{I} &= (\mathbf{L_1D_1L_1^{*}} + \mathbf{L_2D_2L_1^{*}} + \mathbf{L_1D_2^{*}L_2^{*}})^{I} \\ &\quad + \alpha(\mathbf{L_2L_2^{*}})^{I} \\ &\triangleq \mathbf{X}_{AI1} + \alpha\mathbf{X}_{AI2} \end{aligned} \tag{44}$$

3.3 Synthesis under attractive domain constrains

From Theorem 1, it is known that if we find diagonal matrix $\mathbf{S} \leq 0, \mathbf{T} \leq 0$, and suitable parameter matrix $\mathbf{A}$ to satisfy inequalities (13) and (14), then the equilibrium points of CHNN (1) can attract the corrupted patterns in the predetermined neighbor domain $\mathbb{M}(\mathbf{P},\mathbf{Q})$.

Hence, to synthesis the CHNN(1) means to solve the matrix inequalities (13) and (14) . By substitute equilibrium solutions (43) and (44) to inequalities (13) and (14), we get and

Both inequalities (45) and (46) have a form of linear matrix inequalities (LMI), thus the solution procedure will be very easy. With the weight matrix $\mathbf{A}$, the threshold value vector $\mathbf{I}$ can be obtained from (31).

The synthesis procedure is described as follows:

Step 1. Take the given complex-valued memory pattern set $\mathbf{\Lambda} = \{\mathbf{V}^1, \ldots, \mathbf{V}^{m+1}\}$ as outputs of CHNN (1) and calculate the corresponding state vectors $\mathbf{u}^r (r = 1, \ldots, m+1)$ by (26).

Step 2. Check the memory pattern vectors $\{\mathbf{V}^r\}$ and $\{\mathbf{u}^r\}$. If they satisfy the constraint (39), then continue step 3,or else another synthesis method is needed.

Step 3. Calculate $\mathbf{X}_{AR1}$, $\mathbf{X}_{AR2}$, $\mathbf{X}_{AI1}$ and $\mathbf{X}_{AI2}$ by (43) and (44), respectively.

Step 4. For the given attractive domain parameters $\mathbf{P}$ and $\mathbf{Q}$, solve LMIs(45) and (46) and get positive definite diagonal matrix $\mathbf{S}$, $\mathbf{T}$ and variable α.

Step 5. Calculate the complex-valued weight matrix $\mathbf{A}$ by Equation(42 44) and the complex-valued threshold vectors $\mathbf{I}$ by Equation(31).

4. Example

In this section, we give a numerical example to show the effectiveness of our results in this paper. The LMI is solved by the LMI-Toolbox in MATLAB, and the differential equations are calculated numerically via the Runge-Kutta approach with a time step 0.01.

4.1 Selection of CHNN model

In the following example, we design network (1) to store two complex-valued memory vectors and recall test vectors in given attractive domain, where the number of neurons is $n = 3$, time constant $c_i = 2.2 \quad (i = 1,2,3)$, and complex-valued activation function is selected as

$$f_i(u_i) = \tanh(u_i^R) + i\tanh(u_i^I)(i = 1,2,3). \quad (47)$$

It is easy to know that both of the real and imaginary part of such activation function are bounded between -1 and 1, and satisfy local Lipschitz conditions . Here we select the Lipschitz constants as

$$\overline{\mathbf{G}} = diag(\overline{G}_1, \overline{G}_2, \overline{G}_3) = diag(2,2,2),$$

$$\overline{\mathbf{H}} = diag(\overline{H}_1, \overline{H}_2, \overline{H}_3) = diag(2,2,2).$$

4.2 Selection of memory vectors

To choose the memory pattern vectors, three points need to be considered:

1. From activation function (47), all the memory and test vectors must be in set $\mathbf{\Lambda} = \{\mathbf{V} \in C^3, \mathbf{V}^R \in (-1,1)^3, \mathbf{V}^I \in (-1,1)^3\}$.
2. Considering Remark 1, the desired memory pattern vectors V^1 and V^2 must satisfy constraint (39).
3. In order to demonstrate the attractive domain around each memory pattern vector can be large enough, neither the real part nor the imagine part is close to -1 or 1.

According to above consideration, we choose randomly V^1 in set at first. And then we solve the corresponding V^2 to satisfy constrain (39).

4.3 Selection of attractive domain

To decide the parameter matrix $\mathbf{P}$ and $\mathbf{Q}$ of the attractive domain, both the Euclidean distance between the two memory vectors and the definition domain are taken into account.

4.4 A numerical example

Table 1 shows two memory vectors $\mathbf{V}^1$ and $\mathbf{V}^2$ which are specified to be stored in CHNN (1) and the corresponding state vectors $\mathbf{u}^1$ and $\mathbf{u}^2$. From Table 1 we can see that the samples can satisfy constraint (39). Thus we can get Hermitian weight matrix $\mathbf{A}$ by our synthesis scheme.

We design the attractive domain $\mathbb{M}(\mathbf{P},\mathbf{Q})$ of the two vectors by matrix $\mathbf{P}$ and $\mathbf{Q}$, where $\mathbf{P} = diag(11.11, 4, 11.11)$ corresponds to an ellipsoid with radius ,(0.3, 0.5,0.3),$\mathbf{Q} = diag(11.11, 11.11, 25)$ corresponds to an ellipsoid with radius $(0.3, 0.3, 0.2)$.

$$\left\{\begin{array}{ccc} -2\mathbf{PC}+2\overline{\mathbf{G}}\mathbf{S}\overline{\mathbf{G}} & \mathbf{PX_{AR1}}+\alpha\mathbf{PX_{AR2}} & \mathbf{PX_{AI1}}+\alpha\mathbf{PX_{AI2}} \\ * & -\mathbf{S} & 0 \\ * & * & -\mathbf{T} \end{array}\right\}<0 \tag{45}$$

$$\left\{\begin{array}{ccc} -2\mathbf{QC}+2\overline{\mathbf{H}}\mathbf{T}\overline{\mathbf{H}} & \mathbf{QX_{AI1}}+\alpha\mathbf{PX_{AI2}} & \mathbf{PX_{AR1}}+\alpha\mathbf{PX_{AR2}} \\ * & -\mathbf{S} & 0 \\ * & * & -\mathbf{T} \end{array}\right\}<0 \tag{46}$$

Table 1 Memory pattern vectors

$\mathbf{V}^1$	$0.3072+0.1705i$
	$0.3072+0.1705i$
	$-0.5012+0.2083i$
$\mathbf{V}^2$	$-0.3299+0.6813i$
	$-0.1156+0.3795i$
	$0.6439+0.7238i$
$\mathbf{V}=\mathbf{V}^1-\mathbf{V}^2$	$0.6371-0.5108i$
	$0.5283-1.0179i$
	$-1.1451-0.5155i$
$\mathbf{u}^1$	$0.3175+0.1722i$
	$0.4389-0.7555i$
	$-0.5509+0.2114i$
$\mathbf{u}^2$	$-0.3427+0.8315i$
	$-0.1161+0.3995i$
	$0.7648+0.9155i$
$\mathbf{u}=\mathbf{u}^1-\mathbf{u}^2$	$0.6602-0.6593i$
	$0.555-1.1549i$
	$-1.3157-0.7041i$
$\mathbf{V}^*\mathbf{u}=\mathbf{u}^*\mathbf{V}=4.0957$	

By Step3-Step5 in section 3.3, we get

$$\mathbf{S}=diag(221.4173, 221.4173, 221.4173),$$

$$\mathbf{T}=diag(219.2592, 219.2592, 219.2592),$$

weight matrix parameter $\alpha=-1.1382$. The weight matrix $\mathbf{A}$ and threshold vector $\mathbf{I}$ are,

$$\mathbf{A}=\begin{bmatrix} -0.463+0i & 0.8783+0.3314i & -0.3705+1.0122i \\ 0.8783-0.3314i & 0.1626+0i & 0.02+1.4891i \\ -0.3705-1.0122i & 0.02-1.4891i & 0.5557+0i \end{bmatrix}$$

$$\mathbf{I}=\begin{bmatrix} 0.0012-0.2065i \\ -0.105-0.0212i \\ 0.0139-0.1784i \end{bmatrix}$$

Figure 1 shows the attraction of memory vector $\mathbf{V}^1$ and $\mathbf{V}^2$. The dashed ellipses represent the boundaries of the attractive domain of $\mathbf{V}^1$ and $\mathbf{V}^2$. Each solid line in the corresponding ellipse is the output trajectory of CHNN(1) which starts from the ellipse boundary and ends at $\mathbf{V}^1$ or $\mathbf{V}^2$ as time evolution. In Figure 1(a), we choose 8 test vectors whose first element changes on the ellipse in complex plane with the centre being the first element of $\mathbf{V}^1$ or $\mathbf{V}^2$, and the radius of real axis 0.3 and the radius of imaginary axis 0.3, while the other two elements are the same as the corresponding $\mathbf{V}^1$ or $\mathbf{V}^2$. Similarly, Figure 1(b) and 1(c) demonstrate attraction of the second and the third element of $\mathbf{V}^1$ and $\mathbf{V}^2$

As counter-example, Figure 2 shows the change of attraction in the third element of $\mathbf{V}^1$ and $\mathbf{V}^2$ with the change of the free factor α in the connection weight matrix $\mathbf{A}$. From Figure 2, we can see that if the weight matrix $\mathbf{A}$ is choosed unsuitable, the attraction can not be guaranteed.

5. Conclusions

This paper proposes a designation scheme for the complex-valued Hopfield neural network used as associative memory. Both equilibrium constraints and stable conditions are considered in our designation. The equilibrium equation is solved by singular value decomposition. The stable conditions can be transformed to a feasible solution problem in LMI, where the parameter matrix of attractive domain is contained. Thus guarantee the associative memory for the disturbed patterns in the given attractive domain. However, qualitative analysis of the relationship between the free submatrix or variable of weight matrix and the attraction of the CHNN needs for further research.

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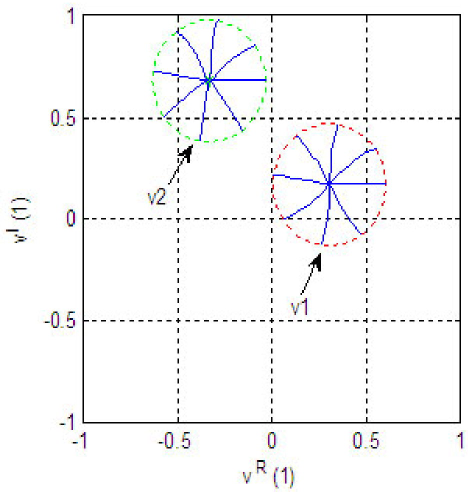
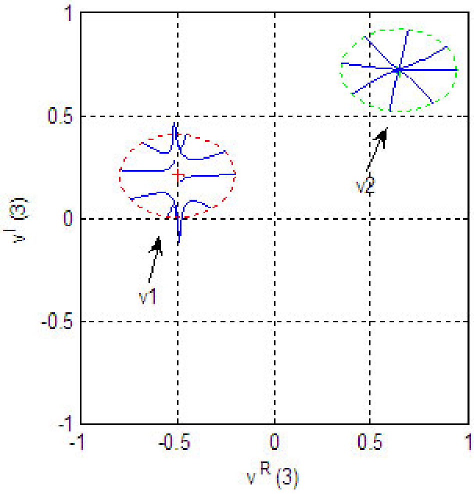
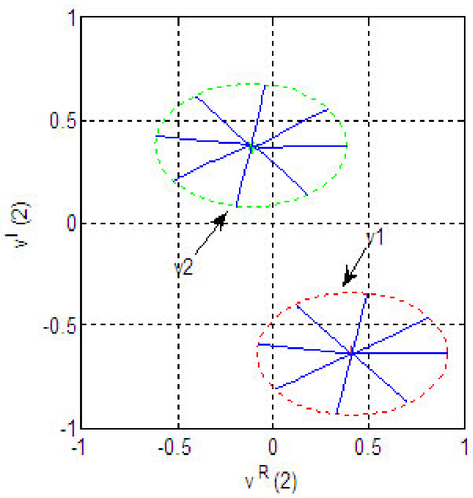


(a) the first element of V^1 and V^2

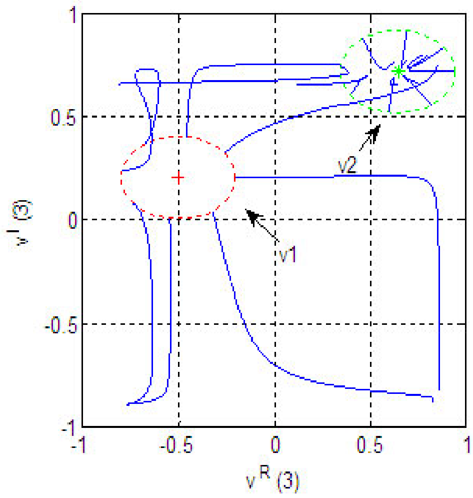


(b) the second element of V^1 and V^2

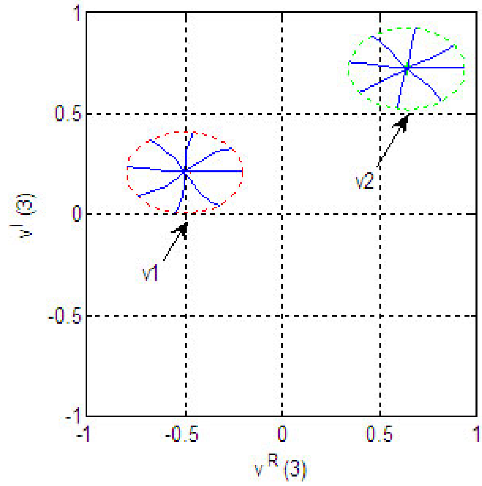


(c) the third element of V^1 and V^2

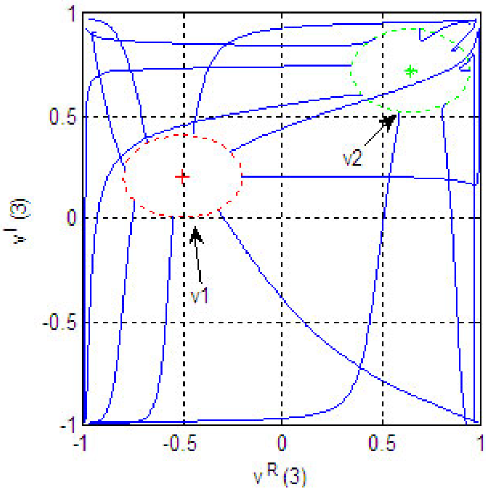
Figure 1 Attraction of V^1 and V^2 ($\alpha = -1.1382$)



(a) $\alpha = 1$



(b) $alpha = 3$



(c) $alpha = 5$

Figure 2 Attraction change with α

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